Basics of the Wheatstone bridge circuit

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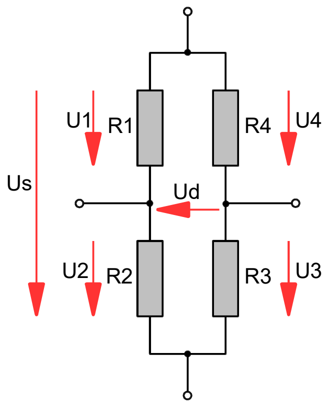
Derivation of the bridge equation

The bridge circuit consists of two voltage dividers connected in parallel. Both voltage dividers are supplied with the bridge supply voltage Us from a common voltage source. The diagonal voltage between the voltage dividers is referred to as the differential voltage Ud.



By applying Ohm's law, you get:

$$\frac{U1}{U2}=\frac{R1}{R2}; \quad \frac{U4}{U3}=\frac{R4}{R3} \qquad \text{eq. 1}$$

By applying the stitch rule you get:

$$U1+U2=Us; \quad U4+U3=Us \qquad \text{eq. 2}$$

Substituting eq. 2 into eq. 1 gives

$$\frac{Us}{R1+R2}=\frac{U1}{R1}; \quad \frac{Us}{R3+R4}=\frac{U4}{R4}; \qquad \text{eq. 3}$$

The differential voltage Ud is obtained by applying the mesh rule:

$$U1-Ud-U4 = 0 \qquad \text{eq. 4}$$

The bridge equation is obtained by substituting eq. 4 into eq. 3:

$$\frac{Ud}{Us} = \frac{R1}{R1+R2} - \frac{R4}{R3+R4} = f(R1, R2, R3, R4) \qquad \text{eq. 5}$$

The linearised from of the bridge equation is obtained by applying the total differential:[1]

$$\frac{\Delta Ud}{Us} = \frac{\partial f}{\partial R1}\cdot\Delta R1 + \frac{\partial f}{\partial R2}\cdot\Delta R2 + \frac{\partial f}{\partial R3}\cdot\Delta R3 + \frac{\partial f}{\partial R4}\cdot\Delta R4 \qquad \text{eq. 6}$$

1Alternatively, but very laboriously, one arrives at the linearised form of the bridge equation by bringing the two terms in Eq. 5 to a common denominator for the subtraction, Ri+ΔRi is written, everything is multiplied out, and all quadratic terms $^{\Delta Ri2}$ are then neglected...

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$$\frac{\Delta Ud}{Us} = \frac{\Delta R1}{R1} - \frac{\Delta R2}{R2} + \frac{\Delta R3}{R3} - \frac{\Delta R4}{R4} \quad \text{eq. 7}$$

Bridge equation for the strain gauge quarter bridge

For stress analysis with strain gauges and for shunt calibration, it is interesting to know how large the error is by applying the linearised bridge equation.

The exact solution is derived below and the error is quantified,

that results from the application of the linearised bridge equation.

The following conditions apply when analysing stress with strain gauges:

$$R1 = R + \Delta R; \quad R2 = R3 = R4 = R \quad \text{Gl 8}$$

By substituting eq. 8 into eq. 5 you get :

$$\frac{Ud}{Us} = \frac{R + \Delta R}{2R + \Delta R} - \frac{1}{2} \quad \text{eq. 9}$$

By expanding the two terms to a common denominator:

$$\frac{Ud}{Us} = \frac{\Delta R}{4R + 2\Delta R} = \frac{\Delta R}{R} \cdot \frac{1}{4 + 2\frac{\Delta R}{R}} = \frac{1}{4}\frac{\Delta R}{R} \cdot \frac{1}{1 + \frac{\Delta R}{2R}} \quad \text{eq. 10}$$

The non-linear component in the equation for the strain gauge quarter bridge is:

$$\frac{1}{1 + \frac{\Delta R}{2R}} \quad \text{eq. 11}$$

The bridge detuning calculated with the linearised bridge equation is too large.

In the stress analysis with strain gauges, the opposite question arises:

The change in resistance (or strain) calculated from the measured bridge detuning is too small.

In practice, the bridge detuning is measured in order to calculate the change in resistance (or strain).

The exact solution for the strain gauge quarter bridge is derived below:

Calculating the strain from the measured bridge detuning

From equation 10, the following is obtained by rearranging:

$$(4R+2\Delta R)\cdot\frac{Ud}{Us} = \Delta R;$$

$$4\frac{Ud}{Us}+\frac{\Delta R}{R}(2\frac{Ud}{Us}-1) = 0;$$

$$\frac{\Delta R}{R}(1-2\frac{Ud}{Us}) = 4\frac{Ud}{Us}; \qquad \text{eq. 12}$$

$$\frac{\Delta R}{R} = 4\frac{Ud}{Us}\cdot\frac{1}{1-2\frac{Ud}{Us}}$$

The non-linear component for the strain gauge quarter bridge is shown as a correction factor in equation 13. The strain determined using the linear equations must be multiplied by the correction factor in order to obtain the exact solution.

$$c=\frac{1}{1-2\frac{Ud}{Us}} \qquad \text{eq. 13}$$

	Linearized solution		exact solution
Ud/Us in mV/V	**Ɛ in µm/m**	**correction factor „c“**	**Ɛ1 in µm/m**
0,005	10	1,00001	10,0001
0,01	20	1,00002	20,0004
0,02	40	1,00004	40,0016
0,05	100	1,00010	100,0100
0,1	200	1,00020	200,0400
0,2	400	1,00040	400,1601
0,5	1000	1,00100	1001,0010
1	2000	1,00200	2004,0080
2	4000	1,00402	4016,0643
5	10000	1,01010	10101,0101

Table 1: Correction factor c as a function of the linear strain or the measured bridge detuning

The correction factor can be shown as a function of the measured bridge detuning. If a k-factor of 2.0 is used as a basis, a corresponding strain can also be shown for orientation.

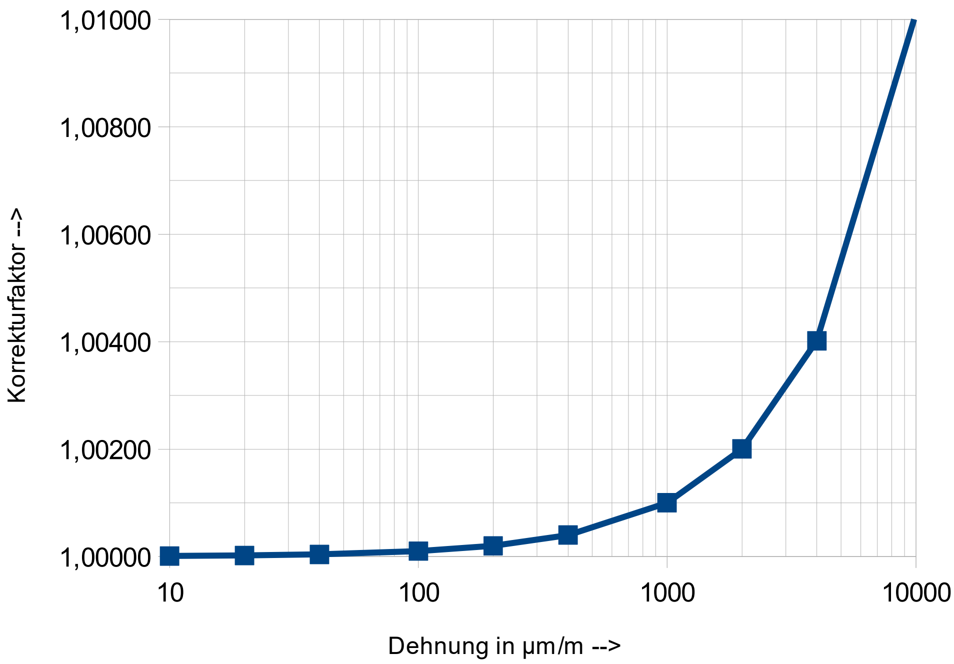

Figure 1: Correction factor as a function of (linearised) strain.

For a quarter bridge calculated using the linear equations for a quarter bridge with k-factor elongation of 1000 µm/m, the actual elongation is 1001 µm/m.

Equations for determining the shunt resistance

The parallel connection of a shunt resistor Rp to one of the four bridge resistors R results in a change in resistance ΔR:

$$\Delta R = \frac{R \cdot Rp}{R+Rp} - R \quad \text{eq. 14}$$

Converted as a relative change in resistance, the result is

$$\frac{\Delta R}{R} = \frac{Rp}{R+Rp} - 1$$
$$\frac{\Delta R}{R} = -\frac{R}{R+Rp} \quad \text{eq. 15}$$

if eq. 15 is inserted into eq. 12 and the correction factor c from eq. 13 is used for the non-linear component, the result is:

$$\frac{R}{R+Rp} = 4\frac{Ud}{Us}\cdot c$$

$$\frac{R+Rp}{R} = \frac{1}{4}\cdot\frac{1}{c}\cdot\frac{1}{\frac{Ud}{Us}}$$

$$Rp = R \cdot \left(\frac{1}{4} \cdot \frac{1}{\frac{Ud}{Us}} \cdot \frac{1}{c} - 1\right)$$

eq. 16

Using equation 16, it is now possible to determine the required shunt resistance as a function of the desired bridge detuning Ud/Us If the term 1/c is taken into account, the exact solution is obtained; without the term 1/c, the linearised solution is obtained.

Selection of shunt resistors

Table 1 shows that a shunt resistor of 86975 ohms causes a bridge detuning of 1,000 mV/V.

			Linearised solution	exact solution
Ud/Us in mV/V	**correction factor „c“**		**Rp in Ohm**	**Rp1 in Ohm**
0,005	1,00001		17499650	17499475
0,01	1,00002		8749650	8749475
0,02	1,00004		4374650	4374475
0,05	1,00010		1749650	1749475
0,1	1,00020		874650	874475
0,2	1,00040		437150	436975
0,5	1,00100		174650	174475
1	1,00200		**87150**	**86975**
2	1,00402		43400	43225
5	1,01010		17150	16975

Table 1: Shunt resistance Rp as a function of bridge detuning (eq. 16, 13) (k-factor = 2, bridge resistance = 350 Ohm)

Table 1 shows that a shunt resistance of 87150 Ohm simulates a strain of 2000 µm/m and causes a bridge detuning of 0.998 mV/V.

exact solution	Linearised solution		Linearised solution	exact solution
Ɛ1 in µm/m	**Ɛ in µm/m**	**Ud/Us in mV/V**	**Rp in Ohm**	**Rp1 in Ohm**
10	9,9999	0,00499995	17499825	17499650
20	19,9996	0,0099998	8749825	8749650
40	39,9984	0,0199992	4374825	4374650
100	99,99	0,049995	1749825	1749650
200	199,96	0,09998	874825	874650
400	399,84	0,19992	437325	437150
1000	999	0,4995	174825	174650
2000	1996	**0,998**	**87325**	**87150**
4000	3984	1,992	43576	43400
10000	9900	4,95	17327	17150

Table 1: Shunt-resistance Rp and strains Ɛ and Ɛ1 as a function of the bridge detuning (eq. 16, 13, 12). (k-factor = 2, bridge resistance = 350 Ohm)